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Mathematical Phenomenology for Thrust-Vectoring-Induced Agility Comparisons

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The recent introduction of thrust-vectoring (TV) maneuverability/controllability into fighter aircraft design methodologies requires reassessment of aircraft equations of motion, especially in the deep poststall (PST) domain. Therefore, a mathematical phenomenology has been developed in this article to assess the main components which affect TV-induced agility and PST-maneuverability. This article identifies the TV-induced forces and moments required to maximize TV control power. It then presents a number of simplified approximate equations for assessing maximized standard agility comparison maneuvers (SACOM) of separate pitch, yaw, and roll TV-induced reversal maneuvers. Such SACOMs are required to compare the performance of future different designs of tailless vectored fighters.

Nomenclature

b = reference span, m
C_D = drag coefficient, dimensionless
$\bar{c}$ = reference mean aerodynamic chord, m
c.g. = center of gravity, % mean aerodynamic chord
C_{fg} = engine nozzle thrust coefficient, its value varies with the jet-deflection angles and the nozzle pressure ratio (which include the effects of throttle angle, Mach number, altitude, etc.) dimensionless [cf. Eqs. (16–18)]
C_{jTV} = thrust-vectoring moment/force terms which vary with the type of the TV-SACOM, force/deg or rad or moment/deg or rad; $j = z, n, L, l$
C_L = lift coefficient, dimensionless
C_l = rolling moment coefficient, dimensionless
$C_{l\beta}$ = rolling moment derivative with respect to sideslip angle, 1/rad
$C_{l\delta a}$ = aileron effectiveness derivative, 1/rad
$C_{l\delta e}$ = stabilator effectiveness derivative, 1/rad
$C_{l\delta\Delta e}$ = differential stabilator effectiveness derivative, 1/rad
$C_{l\delta r}$ = rudder effectiveness derivative (variation of rolling moment coefficient with respect to rudder angle), 1/rad
C_{lp} = roll damping derivative, 1/rad
C_{lr} = rolling moment derivative with respect to yaw rate, 1/rad
C_m = pitching moment coefficient, dimensionless
C_{mo} = basic pitching moment coefficient, dimensionless
C_{mq} = pitching moment derivative with respect to pitch rate, 1/rad
C_n = yawing moment coefficient, dimensionless
$C_{n\beta}$ = yawing moment derivative with respect to sideslip angle, 1/rad
$C_{n\beta^*}$ = yawing moment derivative high angle-of-attack increment with respect to sideslip angle, 1/rad
$C_{n\delta a}$ = yawing moment derivative with respect to aileron deflection, 1/rad
$C_{n\delta e}$ = yawing moment derivative with respect to stabilator deflection, 1/rad
$C_{n\delta\Delta e}$ = yawing moment derivative with respect to differential stabilator deflection, 1/rad
$C_{n\delta r}$ = rudder effectiveness derivative (variation of yawing moment coefficient with respect to rudder angle), 1/rad
C_{np} = yawing moment derivative with respect to roll rate, 1/rad
C_{nr} = yaw damping derivative, 1/rad
C_{py} = side center-of-pressure (for PSM in the y direction)
C_x = longitudinal force coefficient, dimensionless
C_y = side force coefficient, dimensionless
$C_{y\beta}$ = side force derivative with respect to sideslip angle, 1/rad
$C_{y\beta^*}$ = asymmetric side force derivative high angle-of-attack increment with respect to sideslip angle, 1/rad
$C_{y\delta a}$ = side force derivative with respect to aileron deflection, 1/rad
$C_{y\delta e}$ = side force derivative with respect to stabilator deflection, 1/rad
$C_{y\delta\Delta e}$ = side force derivative with respect to differential stabilator deflection, 1/rad
$C_{y\delta r}$ = side force derivative with respect to rudder deflection, 1/rad
C_{yp} = side force derivative with respect to roll rate, 1/rad
C_{yr} = side force derivative with respect to yaw rate, 1/rad
C_z = normal force coefficient, dimensionless
D = the distance from TV nozzle exit to aircraft C_{py}, m
D^* = the distance from TV nozzle exit to aircraft c.g., m
D_{cpy} = the drag operating @ C_{py}, kgf
g = gravitational constant, m/s^2
I_X = moment of inertia about the roll axis, kg-m^2
I_{XY} = cross product of inertia between roll and pitch axes, kg-m^2
I_{XZ} = cross product of inertia between roll and yaw axes, kg-m^2
I_Y = moment of inertia about the pitch axis, kg-m^2
I_Z = moment of inertia about the yaw axis, kg-m^2

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M = aircraft mass, kg
N_i = dimensionless numbers, $i = 1, 2, 3, \ldots$
p = roll rate, rad/s
q = pitch rate, rad/s
$\bar{q}$ = dynamic pressure, N/m^2
r = yaw rate, rad/s
s = reference area, m^2
T = actual (net) thrust [cf. Eqs. (16–18)], kgf
T_i = ideal isentropic (net) thrust [cf. Eqs. (16–18)], kgf
T_v = vertical (pitch) thrust-vectoring component (identical with T_z), kgf
$T_{x,y,z}$ = thrust-vectored components in the (body-axis) x, y, z directions [cf. Eqs. (16–18)], kgf
t = time
V = aircraft true airspeed, m/s
Y = the distance from aircraft centerline to (split-type) TV nozzle centerline, m
Z_{offset} = thrust offset, m
α = angle of attack, also AoA, deg, or rad
β = angle of sideslip, deg, or rad
δ_a = aileron surface deflection (may be a differential angle), deg, or rad
δ_e = elevator (stabilator) surface deflection, deg or rad
$\delta_{\Delta e}$ = differential elevator surface deflection, deg or rad
δ_r = rudder surface deflection, deg or rad
δ_{TV} = effective deflection angle of the jet during pitch and/or yaw thrust vectoring (may be a differential angle during a TV-roll command), deg or rad
δ_{TVD} = effective, differential TV-nozzle/jet deflection during TV-roll-command, deg or rad
δ_v = effective pitch thrust-vectoring angle (may be a differential angle during a TV-roll-command), deg or rad
δ_y = effective yaw thrust-vectoring angle (may be a differential angle during a PSM-yaw-command), deg or rad
θ = pitch angle, deg
ϕ = bank angle, deg
ψ = heading angle, deg

Introduction

TO enhance flight-control power and agility during low-speed, deep poststall (PST), defensive or offensive combat maneuvers, a future pilot may use pitch, yaw-pitch, or pitch-yaw-roll thrust-vectoring control (TVC).[1,2] However, the very definition of agility, especially for maximizing PST-TV maneuverability/controllability, is still a highly debated subject.

Reviews of various agility definitions and of the complex problems involved in generating proper standard agility comparison maneuvers (SACOM) for PST-TV-induced maneuverability and controllability is qualitatively treated in Ref. 3. Reference 3 stresses the fact demonstrated recently by our laboratory and flight tests of PST-TVC F-22, F-15, and F-16 models[1–3,20] that maximum jet-deflection rate is required during pitch, yaw, and roll (TV-induced) maneuver reversals.

An example of such a TV-induced reversal is depicted in Fig. 1 for positive and negative TV-induced "Cobra" maneuvers (see below). While the qualitative definitions of such maneuvers are provided in Ref. 3, this article presents the designers of PST-TVC aircraft with a mathematical phenomenology for treating multifunctional thrust-vectoring-induced maneuverability under PST-SACOM conditions.

Conventional Limitations

The mathematical techniques used to estimate aircraft characteristics during PST maneuvers are becoming increasingly complex as the angle of attack (AoA) is increased, especially in the deep PST domain beyond about 80 deg.[3–6] Thus, at the present time it may not be practical to extract PST-TV coupling coefficients, stability, and control derivatives from

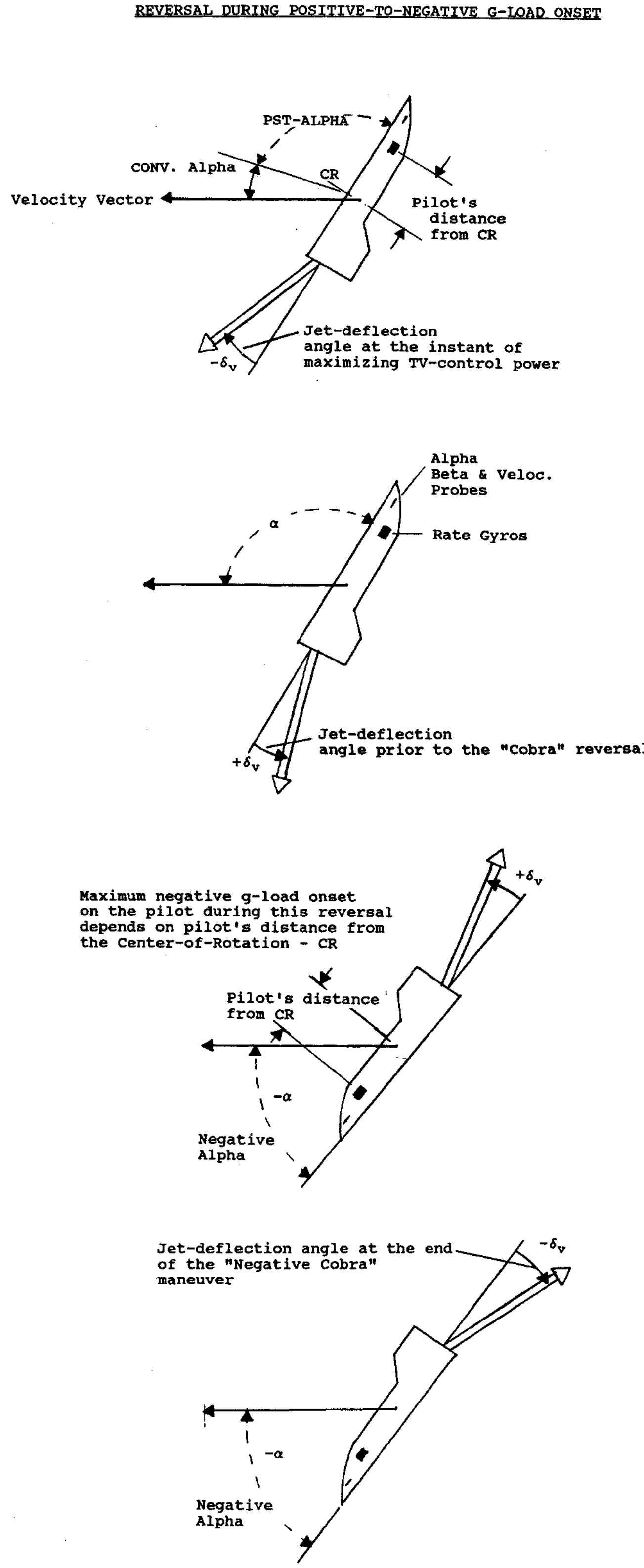

Fig. 1 Thrust-vectoring control power required during Cobra maneuvers.

conventional mathematical phenomenology. Nevertheless, as attempted below, one may add proper TV terms into conventional phenomenology and then try to extract physical insight and new guidelines under the restrictions imposed by SACOMs, and, accordingly, by a set of simplifying assumptions. These assumptions take into account the limitations and needs posed by PST-TV.

Following our recent jet propulsion laboratory/flight tests of powered TV models, it was concluded that maximization of TV control power requires not only the maximization of turn rates but also the minimization of the time to perform TV-induced, turn-rate reversals.[3]

For instance, to rapidly acquire a target and then lock-track it with precision, unload, or switch to the next target, one needs the capability to rapidly stop the high nose-turn-rate, or even reverse it. Maximization of TV control power is therefore required at the point of reversal onset. Hence, maximization of PST-TV induced agility requires analysis of the jet deflection reversal power needs during rapid PST-reversal onsets, or "rapid rotational whipping" (see below). For this purpose a new set of SACOM reversals has been introduced in Ref. 3, including a well-defined set of initial and end SACOM conditions. The newly proposed SACOM reversals include pitch, yaw, and roll reversals by way of conventional, TVC, or combined flight control means. Moreover, mathematical definitions of TV-induced moments, forces, and accelerations are also needed now for computer and large-scale centrifuge simulations of pilot tolerances during rapid PST-TV reversals/onsets.

A mathematical phenomenology has therefore been developed in this article to assess the main components which affect TV-induced agility and PST maneuverability.

First identified are TV forces and moments required to maximize deep poststall yaw-pitch-roll TVC power. It then presents a number of simplified approximate equations for assessing SACOM of separate pitch, yaw, and roll TV-reversal maneuvers. This phenomenology is required in any attempt to maximize yaw-roll-pitch TV-induced, PST agility parameters.

Proposed Mathematical Phenomenology

The phenomenology presented below is characterized by the bold assumption that to describe the aerodynamic behavior of PST-TVC aircraft one may still use the first-order partial derivatives as an approximation. Thus, the 6 degree-of-freedom equations of motion with the yet unspecified thrust-vectoring terms are

$$\dot{\alpha} = q + \{-[\bar{q}sC_x/MV - (g/V)\sin\theta + r\sin\beta]\sin\alpha + [\bar{q}sC_z/MV + (g/V)\cos\theta\cos\phi - p\sin\beta]\cos\alpha\}\sec\beta \tag{1}$$

$$\dot{\beta} = -\{[\bar{q}sC_x/MV - (g/V)\sin\theta]\sin\beta + r\}\cos\alpha + [\bar{q}sC_y/MV + (g/V)\cos\theta\sin\phi]\cos\beta - \{[\bar{q}sC_z/MV + (g/V)\cos\theta\cos\phi]\sin\beta - p\}\sin\alpha \tag{2}$$

$$\dot{p} = \{-[(I_z - I_y)/I_x + I_{xz}^2/I_xI_z]qr + [1 - (I_y - I_x)/I_z]I_{xz}pq/I_x + \bar{q}sb/I_x[C_l + I_{xz}C_n/I_z]\}/(1 - I_{xz}^2/I_xI_z) \tag{3}$$

$$\dot{q} = \bar{q}s\bar{c}C_m/I_y + [(I_z - I_x)/I_y]pr + I_{xz}(r^2 - p^2)/I_y \tag{4}$$

$$\dot{r} = \{[I_{xz}^2/I_xI_y - (I_y - I_x)/I_z]pq - [1 + (I_z - I_y)/I_x](I_{xz}/I_z)qr + (\bar{q}sb/I_z)[(I_{xz}/I_x)C_l + C_n]\}/(1 - I_{xz}^2/I_xI_z) \tag{5}$$

$$\dot{V}/V = [\bar{q}sC_x/MV - (g/V)\sin\theta]\cos\alpha\cos\beta + [\bar{q}sC_y/MV + (g/V)\cos\theta\sin\phi]\sin\beta + [\bar{q}sC_z/MV + (g/V)\cos\theta\cos\phi]\sin\alpha\cos\beta \tag{6}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{7}$$

$$\dot{\phi} = p + r\cos\phi\tan\theta + q\sin\phi\tan\theta \tag{8}$$

$$\dot{\psi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta \tag{9}$$

$$C_x = C_L(\alpha, \delta_e)\sin\alpha - C_D(\alpha, \delta_e)\cos\alpha\cos\beta + T_x/\bar{q}s \tag{10}$$

$$C_Y = C_Y(\alpha, |\beta|, \delta_e) + C_{Y\delta a}(\alpha)\delta_a + C_{Y\delta r}(\alpha)\delta_r + (b/2V)[C_y(\alpha)r + C_{yp}(\alpha)p] + \Delta C_{Y\beta^*}(\alpha, \beta) + C_{y\delta\Delta e}(\alpha, \delta_e)\delta_{\Delta e} + T_y/\bar{q}s \tag{11}$$

$$C_z = -[C_L(\alpha, \delta_e)\cos\alpha + C_D(\alpha, \delta_e)\sin\alpha\cos\beta] + C_{(z\mathrm{SC})}\delta_{\mathrm{TV}} + T_v/\bar{q}s \tag{12}$$

$$C_l = C_{l\beta}(\alpha, |\beta|)\beta + C_{l\delta a}(\alpha, \delta_e)\delta_a + C_{l\delta r}(\alpha, |\delta_r|)\delta_r + (b/2V)[C_{lp}(\alpha)p + C_{lr}(\alpha)r] + C_{l\delta\Delta e}(\alpha, \delta_e)\delta_{\Delta e} + \Delta C_{l\beta}(\alpha, \beta) + C_{l\mathrm{TV}}\delta_{\mathrm{TV}} \tag{13}$$

$$C_m = C_{mo}(\alpha, \delta_e) + (c/2V)C_{mq}(\alpha)q + T(\Delta Z_{\mathrm{offset}})/\bar{q}s\bar{c} + C_{m\mathrm{SC}}\delta_{\mathrm{TV}} + C_{m\mathrm{TV}}\delta_{\mathrm{TV}} \tag{14}$$

$$C_n = C_{n\beta}(\alpha, \beta, \delta_e)\beta + C_{n\delta a}(\alpha)\delta_a + C_{n\beta}(\alpha, \beta) + C_{n\delta r}(\alpha, \beta, \delta_r, \delta_e)\delta_r + (\bar{c}/2V)[C_{np}(\alpha)p + C_{nr}(\alpha)r] + C_{n\delta\Delta e}(\alpha, \delta_{\Delta e})\delta_{\Delta e} + \Delta C_{n\beta}(\alpha, \beta) + \Delta C_{n\beta^*}(\alpha, \beta) + C_{n\mathrm{TV}}\delta_{\mathrm{TV}} \tag{15}$$

$$T_x = C_{fg}T_i\cos\delta_v\cos\delta_y \tag{16}$$

$$T_v = C_{fg}T_i\sin\delta_v\cos\delta_y = T_z \tag{17}$$

$$T_y = C_{fg}T_i\cos\delta_v\sin\delta_y \tag{18}$$

This set of 18 equations completes our simplified phenomenology for thrust-vectoring-induced maneuvers.

General Thrust-Vectoring Terms

Only linear expansions of moments and forces have been employed, including the unspecified scalar thrust-vectoring (TV) notation for thrust-vectoring-induced supercirculation (zSC) in Eqs. (12) and (14) (for definitions and physico-aerodynamic fundamentals see Ref. 1). This phenomenology is based on effective jet deflection angles δ_v and δ_y, or, in general, on the universal scalar δ_{TV}. For instance, $C_{m\mathrm{TV}}$ denotes the dimensionless pitching moment per radian of effective jet deflection in the pitch coordinates. Similarly $C_{l\mathrm{TV}}$ denotes the roll-thrust-vectoring moment per radian due to differential jet deflection in (split-type) single or S-type twin-engine nozzle(s).[1]

The T_x, T_v, and T_y terms which appear in Eqs. (10–12) denote the direct effective thrust-vectoring forces in the x, z, and y (body-axis) directions, respectively, as defined by Eqs. (16–18). No constraints are placed on the effective angles through which the thrust is vectored in order to allow as much range of thrust vectoring as would be needed during short takeoff and landing (STOL), air-to-air combat, or air-to-ground attack.

Effective jet deflections should not be confused with geometric TV deflection angles of pitch-flaps and yaw-vanes inside two-dimensional, converging-diverging (2D-CD) nozzles, or with deflection-angles of external paddles, or of axisymmetric multifunction TV nozzle(s). The former nozzles are expected to provide the fastest available responses to flight-control inputs, mainly due to the inherent faster rate by which simple yaw/pitch thrust-vectoring vanes/flaps can be rotated in comparison with the complex yaw/pitch deflections of multiflap sliding rods/rings mechanisms of axisymmetric TV nozzles.

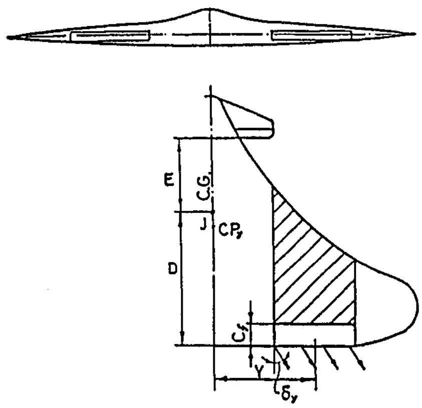

Fig. 2 High-aspect ratio, roll-yaw-pitch, thrust-vectoring nozzles (upper drawing) characterize vectored aircraft. Increasing the roll arm Y increases the roll moment. This increase is expected to be most critical at high angles of attack. The shaded area represents supercirculation affected area.[1]

Thus, to maximize TVC and combat effectiveness we stress the need for yaw-pitch or yaw-pitch-roll 2D-CD nozzles.[1,2,3,20,21] Of cardinal importance to maximize TV-roll agility during high AoA maneuvers is Y (Fig. 2).

The dimensionless C_m equation contains two terms associated with pitch thrust-vectoring: 1) $C_{mSC}\delta_{TV}$ and 2) $C_{mTV}\delta_{TV}$. The first is directly affected by $\bar{q}s$, the external aerodynamic flow regime, whereas, the $\bar{q}s$ effect on the thrust-vectoring moment $C_{mTV}\delta_{TV}$ is different and much less pronounced, especially for PST-vectorable engine inlets which resist distortion effects at compressor inlet at high angles of attack. Therefore, except for the "supercirculation" terms in Eqs. (12) and (14), all other TV terms can be treated irrespective of the dynamic pressure $\bar{q}s$. During pure thrust-vectoring the dynamic pressure term which multiplies C_m in Eq. (4) is cancelled out by the use of proper units such as those in Eq. (12). However, only high-aspect ratio nozzles that are well-integrated with the wing trailing edge (Fig. 2) increase lift (supercirculation) during jet-down deflections.

Applicability Restrictions

There are two types of coupling: 1) kinematic and 2) aerodynamic. The coupling terms cannot be neglected in an exact analysis of PST-TV flight unless some simplified, decoupled-SACOMs are made (see below). For instance, due to separated flows and stalling effects, PST-TVC aircraft flying at AoA > 60 deg exhibit strong aerodynamic/propulsion coupling.[20] It has been recently demonstrated during our flight tests with $\frac{1}{7}$-scale PST-TVC F-15 models, that a pure TVC-yaw command produces a strong TV-induced roll, depending on the size of the vertical stabilator. However, depending on the particular tailless TVC aircraft design,[1,20] and on the particular SACOM,[3] these effects can be minimized or neglected in a preliminary analysis of PST-TVC-SACOM.

Parameters which are not listed here include the Mach number, altitude, and gyroscopic effects.[5,8,20] However, their effect partially enters the phenomenology through the effect of nozzle pressure ratio (NPR) on C_{fg}, etc.[1] A low-speed SACOM is also assumed (i.e., $M = <0.6$). Therefore, the flight can be assumed to be in the incompressible flow regime. Various other effects have been neglected in this model. For instance, the asymmetric effects due to thrust, fuel distribution, and aerodynamics have been neglected, as well as engine turbulence noise (cf. Refs. 3 and 4 and below).

In assessing this phenomenology we also note:

1) Various eight-state ($\alpha, \beta, p, q, r, \theta, \phi, V$) aircraft models are available in the literature (cf., Refs. 5–19).

2) The present deterministic phenomenology must be further modified by the presence of a superimposed spectral density of the TV-SACOM measurement noise, especially when flying our low-weight/low moment-of-inertia scaled models.[1,3,6,11,18,19] Available "stochastic-statistical" methods may then be combined with a standard spectrum for atmospheric turbulence and with "maximum likelihood estimation concepts."[5,6,10,13]

3) Cross-coupling terms are normally not included in the analyses when the flight data are gathered during stabilized flight at low AoA. These terms are needed when the aircraft is expected to have aerodynamic cross coupling between the longitudinal and lateral-directional aerodynamic modes.

4) Equations (1–6) can be divided into two sets: 1) the longitudinal and 2) the lateral-directional equations.

Basic Cobra Reversal SACOM

For a cobra-type, pitch-only, "pseudohorizontal," PST-TV-SACOM,[2,3,20] performed with PVA or with frozen conventional control surfaces, the vehicle is very rapidly whipping the air. Up and down in positive and down and up in negative Cobra-type maneuvers. This rapid nose-pointing, or "rotational whipping/onset" capability may keep the aircraft's flight path approximately horizontal for a short duration. Depending on the thrust-to-weight ratio, Mach number, altitude, and the combat scenario,[1,20] as well as on the rotational whipping/onset duration, the aircraft may gain some altitude prior to reversing this trend during a positive Cobra maneuver, whereas, the trend is only downward during a negative maneuver (Fig. 1).

To gain a preliminary understanding of the main parameters which affect this SACOM, we assume that during the maneuver the flight path remains at a "pseudo-constant-altitude." Hence, we further assume that under these conditions the ϕ, β, p, $\dot{p}$, r, $\dot{r}$, $\dot{\beta}$, δ_e, δ_a, δ_r, $\delta_{\Delta e}$, C_l, C_n, and C_Y terms vanish, while $\theta \cong \alpha$ and $\dot{\alpha} \cong q$. Moreover, the supercirculation term can be neglected for low-aspect ratio TV nozzles, e.g., as is the case with some of our early $\frac{1}{7}$-scale PST-TV F-15 flying models. This conclusion is due to the low surface area affected by such nozzles.[1,2]

The term $T(\Delta Z_{\text{offset}})$ vanishes when the nozzle(s) thrust acts through the aircraft c.g. This assumption is usually not met in reality. Yet, using conventional control surfaces, the flyer can pretrim the aircraft controls so as to approximate the total effects associated with the aforementioned assumptions for each particular SACOM. Under these conditions, the flyer command is a pure δ_v input, for which the aircraft response in controlled pseudo-constant-altitude, Cobra reversal flight is approximately determined by

$$C_x = C_L(\alpha)\sin\alpha - C_D(\alpha)\cos\alpha + T_x/\bar{q}s \quad (19)$$

$$C_Y = 0 \quad (20)$$

$$C_z = -[C_L(\alpha)\cos\alpha + C_D(\alpha)\sin\alpha] + T_v/\bar{q}s \quad (21)$$

$$C_l = 0 \quad (22)$$

$$C_m = C_{mo}(\alpha) + (C/2V)C_{mg}(\alpha)q + C_{mTV}\delta_v \quad (23)$$

$$C_n = 0 \quad (24)$$

$$Mg = \bar{q}s(C_x \sin\alpha - C_z\cos\alpha) \quad (25)$$

$$\dot{q}I_y = \bar{q}sc[C_{mo}(\alpha) + (C/2VC_{mg}(\alpha)q + C_{mTV}\delta_v] \quad (26)$$

$$M\dot{V} = \bar{q}s(C_x\sin\alpha + C_z\cos\alpha) \quad (27)$$

For this Cobra-type SACOM Eqs. (16–18) reduce to

$$T_x = C_{fg}T_i\cos\delta_v \quad (28)$$

$$T_v = C_{fg}T_i\sin\delta_v \quad (29)$$

$$T_y = 0 \quad (30)$$

Equations (19–30) define our simplified phenomenology for such a "pure" PST-TV-SACOM. Various numerical and analytical solutions of this set (with particular initial and boundary conditions) can be investigated and gradually employed for working back and forth between theory and well-defined flight tests.[20] One of these, and perhaps the most useful one, is treated next.

Jet-Reversing at 90-Deg AoA Cobra SACOM

This particular Cobra SACOM involves reversing the direction of the jet from maximum deflection angle in one direction to the other, during positive or negative Cobra maneuvers at exactly positive or negative 90-deg AoA (while trying to keep the flight path almost horizontal throughout the maneuver).

At AoA = 90 deg (positive or negative), the aerodynamic "lift" vanishes. We then consider only small variations of θ, $\dot{\theta}$, q, $\dot{q}$, etc. around this value. The purpose of making this bold assumption is to examine the main variables which affect the maximization of pure TVC power during such a reversal, in line with the principles set forth in Ref. 3. In practice, however, even the value of C_{fg} varies slightly throughout the PST-TV-induced maneuver.[1,21]

Now, by freezing all conventional control surfaces and by concentrating only on the δ_v command, we obtain a very simple and useful set

$$C_x = C_{fg}T_i \cos \delta_v/\bar{q}s \tag{31}$$

$$C_z = -C_D(90) + C_{fg}T_i \sin \delta_v/\bar{q}s \tag{32}$$

$$C_m = C_{mo}(90) + C_{m\mathrm{TV}}\delta_v \tag{33}$$

$$Mg = C_{fg}T_i \cos \delta_v = T_x \tag{34}$$

$$\dot{q}I_y = \bar{q}sc[C_{mo}(90) + C_{m\mathrm{TV}}\delta_v] \tag{35}$$

$$M\dot{V} = C_{fg}T_i \cos \delta_v = T_x = Mg \tag{36}$$

A positive PST-TV cobra-type SACOM entails two commands: 1) a (δ_v) nose-up command, and 2) when the AoA reaches, e.g., 90 deg, a rapid ($-\delta_v$) nose-down command. These commands reverse the sign of the TV-induced forces and moments. During the evolution of this SACOM they generate rapidly opposing "g onsets/rotational whippings" on the pilot (i.e., from positive to negative g loads during positive Cobra SACOM reversals, and vice versa during the negative ones).

Equation (26) or (35) is further simplified when the pitching moment coefficient C_{mo} can be neglected in comparison with the TV one. Under such SACOM conditions, the pitch rate becomes approximately proportional to the sign of sin δ_v, and is readily measured and compared with the TV command, especially during rapid TV reversal commands.

Reference 3 provides the reasonings for maximizing the TV control power by way of such δ_v pitch reversal commands.

Alternatively, one may examine a slow TV pitch-only SACOM, in which the AoA remains constant. One possibility is to perform a constant AoA climb in the vertical plane so that instead of Eq. (35) one can establish the functional

$$\dot{q} = f[\delta_v(t)] \tag{37}$$

However, this SACOM invalidates the assumption that q and the time derivative of the AoA are approximately equal. Therefore, such a maneuver may hardly serve as a PST-TV-SACOM. Consequently, this phenomenology dictates that PST-TV-SACOMs be performed with only the horizontal Cobra reversal-type onsets at positive and negative AoA.

The maximum range of the nozzle's δ_V jet deflection, and the whipping onset speeds determine this agility component.[3] The fastest whipping onset rate is extractable when the flyer, pilot, or the automatic flight control, commands an approximate Dirac-type reversal function. It is, however, determined also by the inherent fastest rate a TV nozzle can effectively deflect/reverse its jets (see below).

Agility Restricted by Pilot Location?

The effects of negative (pitch) g onsets/whippings on critical physiological functions of the pilot during maximum, low-subsonic, PST-TV Cobra maneuvers, etc. may pose a limitation on PST-TV agility.[20] These effects depend, among other things, on the distance from the pilot's head to the center of rotation during rapid TV maneuvers. However, we do not know yet the location of this center of rotation during rapid g onsets/whippings into the deep PST domains. In fact, we have recently observed during flight tests with subscale TV models, that under certain SACOMs it may move away from the c.g. during rapid, PST-TV SACOMs.[20] Our current studies focus attention on this subject since understanding these new phenomena is highly critical to the potential imposition of certain pilot-induced combat agility limitations, and the search for the optimal location of the pilot's seat in future tailless vectored fighters.

Two-Dimensional or Axi-TV Nozzles?

Is there an inherent technology limit associated with all axisymmetric yaw-pitch TV-nozzles? If one exists, such a technology limit dictates relatively slow, maximum jet-deflection rates, in comparison with jet-deflection rates extractable from yaw vanes installed inside 2D-CD nozzles. (The internal yaw vanes may be uncooled, or internally cooled, as, e.g., advanced first-stage, turbine stators are.[21] Nevertheless, from the point of views of engine/AB-duct/nozzle/actuators and structure/weight/control/cost changes, both nozzles are approximately equal, when low-aspect ratio 2D-CD nozzles are examined.)

All internal yaw vanes which have been developed and laboratory/flight-tested by Gal-Or since 1984, for low- or high-aspect ratio two-dimensional and 2D-CD nozzles, can yaw deflect the jets faster than any complex mechanism of axisymmetric nozzle flaps, rings, ducts, and sliding rodes can.[1-3,20,21] Since such limiting rates are critical in close combat engagements, these methodological differences may incorporate far reaching consequences for figher aircraft design criteria.

Pure Sideslip Maneuvers with Tailless Vectored Fighters

Tailless, pure, or "ideal" thrust-vectored aircraft can perform pure sideslip maneuvers (PSM) with constant (steady state) horizontal heading, without banking.[1-3,20] During such a PSM, one nozzle is employed to deflect its jet in the yaw direction until its vector coincides with the side c.p. C_{py}. This causes PSM zero yawing rate and banking, i.e., r, p, q, and $\dot{\beta}$ vanish, but not β. (To perform this SACOM, the nonyawing, axial thrust generated by the second nozzle is somewhat reduced to equal that left over by the first nozzle, so as to avoid a yawing moment on the TV aircraft.)

Alternatively, the second nozzle yaw deflection potential may be employed for very rapidly yawing the nose of the aircraft (again, without banking), so as to acquire a target with minimal energy dissipation. A similar PST-TV acquisition, on the other hand, dissipates considerably more energy.[1] Therefore, to acquire any target, such a TV-PSM-YAW may be combined with a partial roll.[1] A simplified phenomenology for guiding the design of such SACOMs is provided below.

Consider the simplest model, e.g., we assume that such PSM or PSM-YAW SACOMs are performed at zero AoA and zero-pitch attitude with no banking and roll. For tailless pure vectored aircraft we also assume the dominance of TV forces and moments over the conventional ones (or the absence of conventional flight control means), as well as negligible coupling between TV yaw and TV-induced roll through the tail, etc. Here the α, $\dot{\alpha}$, θ, ϕ, $\dot{p}$, $\dot{q}$, q, r, δ_v, δ_e, δ_a, δ_r, $\delta_{\Delta e}$ $T(\Delta Z_{\text{offset}})$, C_z, C_l, C_m, C_n terms vanish, and from Eqs. (2), (6), (10), and (11) one obtains

$$C_y \cos \beta = C_x \sin \beta \tag{38}$$

$$\dot{V}/V = (\bar{q}s/MV)(C_x \cos \beta + C_y \sin \beta) \tag{39}$$

$$C_x = (C_{fg}T_i \cos \delta_y)/\bar{q}s - C_D[\alpha(0)] \tag{40}$$

$$C_y = C_y(\beta) + (C_{fg}T_i \sin \delta_y)/\bar{q}s \tag{41}$$

$$T_x = C_{fg}T_i \cos \delta_y \tag{42}$$

$$T_v = 0 \tag{43}$$

$$T_y = C_{fg}T_i \sin \delta_y \tag{44}$$

Transient PSM/Yaw-SACOMs

For extracting maximal TV-induced roll and yaw flight control, the pitch and yaw deflections of the jet in each of the two nozzles are independently controlled. Under such yawing conditions each nozzle, or half-nozzle, provides different thrust efficiency, i.e., each may operate with a different C_{fg} value. Therefore, during independent yaw-deflections, Eqs. (40) and (41) with the aforementioned assumptions, become

$$C_x = (C_{fg1}T_{i1} \cos \delta_{y1} + C_{fg2}T_{i2} \cos \delta_{y2})/\bar{q}s - C_D[\alpha(0)] \tag{45}$$

$$C_y = C_y(\beta) + (C_{fg1}T_{i1} \sin \delta_{y1} + C_{fg2}T_{i2} \sin \delta_{y2})/\bar{q}s \tag{46}$$

where the numbers refer to each of the two TV jets/nozzles.

Recalling that during a pure steady-state PSM, the jet of one nozzle is yaw-deflected until its vector coincides with the side c.p., the yaw δ_{y2} of the other nozzle is zero and the axial thrust generated by this nozzle is throttle adjusted to equal that left over by the first nozzle, one concludes that the pure vectored aircraft performs PSM without yaw rate and banking, provided all sums of moments vanish.

A maximum TV-induced yaw rate is extractable when both nozzles direct the jets in the same yaw direction. Yet, maximization of TVC power is demonstratable, as in the previous PST-TV-SACOM, only through a TVC yaw reversal, when a proper δ_y command is performed. Under these conditions one can investigate the maximum rate of change of r dot of two competing aircraft.

A more promising, yet more complicated maneuver is obtainable as follows. During, e.g., a defensive PSM, the jet of the second nozzle is simultaneously yaw deflected for yawing the nose of the aircraft (without banking) so as to acquire a target with minimal energy dissipation. Since a similar PST-TV acquisition dissipates considerably more energy, one may perform a rapid roll, followed by such a TV yaw or PSM/Yaw maneuver, especially in target-rich scenarios.[1-3]

A similar notation may be employed to rewrite the equations for differential TV-pitch maneuvers, e.g., during PST-TV roll commands of tailless TVC aircraft, and especially during TV roll reversal SACOMs at very high AoA.[3] (Note, that it is a "poststall roll around the velocity vector," or a "V-roll," or, as a short, new term proposed here, simply a "voll." Thus, at AoA = 90 deg the voll SACOM transforms into a yaw SACOM. The author is generating a new short term-word here which is highly needed for the new poststall flight domain! It resembles but is not related to the word volley—the flight of a missile, ball, etc. The current alternative is for the pilot to say in combat "Poststall roll around the velocity vector"—this is too long for winning a poststall maneuver.)

Perform a SACOM During Atmospheric Turbulence Under Separated-Flow Conditions

Neither full-scale aircraft, or powered scaled models used in our studies, can avoid flying in atmospheric turbulence. Hence, it is desirable to devise analytical tools that properly extract meaningful engineering conclusions from flight data that have been collected under such conditions.[20] These flight data include a kind of superimposed spectral density of the measurement noise, especially when flying low-weight/low moment-of-inertia scaled models. Available stochastic methods may then be combined with a standard spectrum for atmospheric turbulence to generate maximum likelihood estimation concepts.[6]

Another situation occurs under separated-flow conditions in the PST domain, when the aircraft, or the scaled dynamic model is driven by unknown stochastic inputs.

Unless the separation is mild enough to permit a well-verified mathematical model to approximate the SACOM, little can be done to extract meaningful engineering conclusions under these conditions.

While various ad hoc methods have been devised to overcome these problems for what is currently categorized as "high AoA research," no reliable solution to the problem presently exists beyond approximately 70-deg AoA.

Measurement Difficulties

The very method of measurement affects the results produced by flying models. The combined weights of probes, gyros, batteries, computers, telemetry equipment, servos, wires, and safety devices affect the moments of inertia, stability margin, thrust-to-weight ratio, etc. In turn, the additional masses may be properly used to generate certain preferred similarities between model and full-size vehicles.[1,20] For instance, in comparing the performance of vectored with unvectored models, both should have the same mass, mass distribution, stability margin, thrust-to-weight ratio, drag, etc.

However, the maximum thrust available by all small-scale, two-dimensional TV nozzles is considerably lower than that extractable from similar axisymmetric nozzles. (This is not the case with full-sized nozzles.) Hence, external thrust vectoring, i.e., the vectorable thrust produced by small axisymmetric nozzles that are equipped with variable external paddles (and provide unhindered cruise thrust), have been verified by our flight tests as the optimal choice for performing proper PST-TV-SACOMs.

Serious problems are also posed by the unavailability of PST (vectorable) engine inlets. Moreover, materials, servos, engines, nozzles, cooling means, IFPC, etc. do not scale-up easily by general rules. Therefore, the expected SACOM reference baseline is dictated by technology limits in each of the aforementioned categories. Similar restrictions apply to differences in Reynolds number, turbulence spectrum, propulsion coupling to stability and control derivatives, conventional and TV control effectiveness, as well as to different uncertainties in the measured values of α, β, θ, ϕ, p, r, q, and V at high AoA.

Considerable differences have also been observed when a comparison was made between wind-tunnel estimates of our tailless, $\frac{1}{32}$-scaled, PST-TV models, and our $\frac{1}{7}$-scaled model flight tests. These differences are partially attributable to differences in aerodynamic flow between the static wind-tunnel tests and the dynamic flight maneuvers.

Flight tests of unpowered remotely piloted $\frac{3}{8}$-scaled F-15 model[11,18] have also indicated considerable differences from full-scale F-15's dynamic behavior above an angle-of-attack of 30 deg.

Such differences make PST-TV phenomenology and SACOM tests a very demanding subject. Nevertheless, in line with our flight-testing experiences, the proposed methodology/phenomenology can guide the design of multifunctional PST-TV fighter aircraft and delineate the sources of similarities and differences.

Conclusions

The main thrust-vectoring moments and forces required to maximize deep poststall agility and controllability have been identified, using multifunctional yaw-pitch-roll thrust-vectoring control power. Approximate equations required for assessing separate pitch, yaw and roll thrust-vectoring reversal maneuvers have been assessed in light of new SACOM needs.

The analysis is applicable to PST, PSM, YAW, or PSM-YAW-SACOMs. It may also be employed during flight tests of pure vectored RPVs, or of various PST-TV scaled model upgrades of extant fighter aircraft.

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